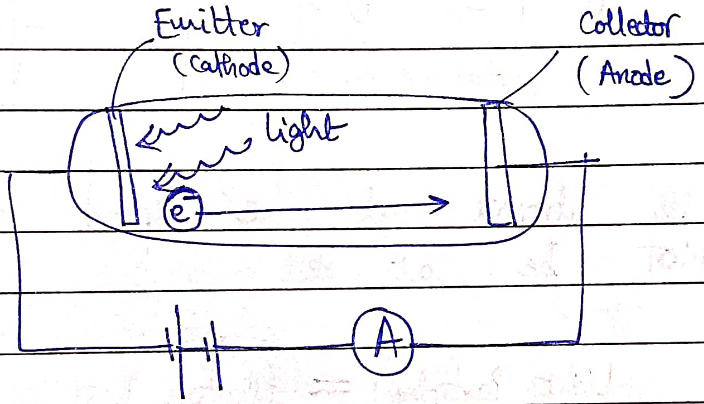
Modern Physics

1) Photoelectric Effect
2) X-rays
3) Bohr's Model
4) Nuclear Physics

Hertz' Experiment



When light shined, current observed.

Observations in Photoelectric effect which do NOT support wave theory —

1) Instantaneous current on shining light.
(as ϕ is min. amt. of energy req. to free e^-s).
(work fx^n)

2) $\lambda \leq \lambda_{max} \Rightarrow e^-$ even on faint intensity light
$\lambda > \lambda_{max} \Rightarrow$ No current no matter what intensity of light.

$\boxed{\lambda_{max} = \text{(Threshold wavelength)}}$ and $\boxed{\nu_{min} = \text{(Cut off frequency)}}$

Quantum Theory of Light

Light Particle : Rest mass $= 0$ | Energy $\neq 0$
Charge $= 0$ | Momentum $\neq 0$

$$\boxed{E = h\nu = \frac{hc}{\lambda}}$$

$$\boxed{p = \frac{E}{c}}$$

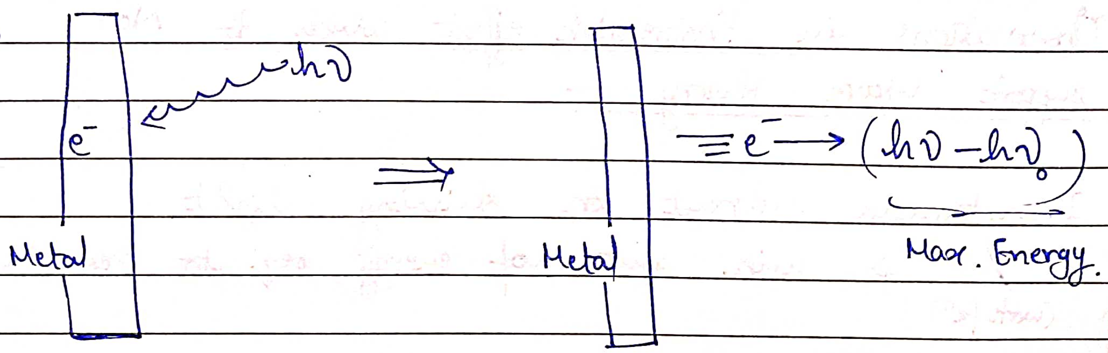
Acc. Special Theory of Relativity,

$$\boxed{m = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}$$

Light Particles travel at 'c' speed and can NOT be at rest

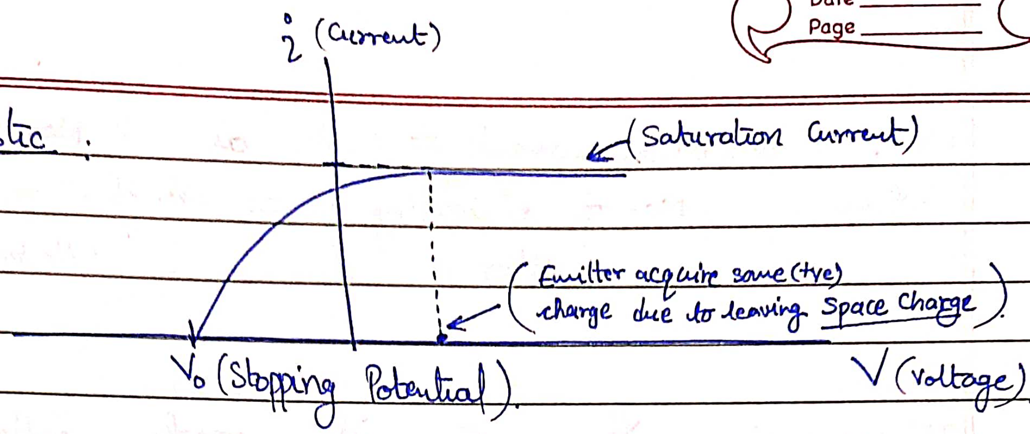
Nowadays, (Light Particles) = Photon.

Now,

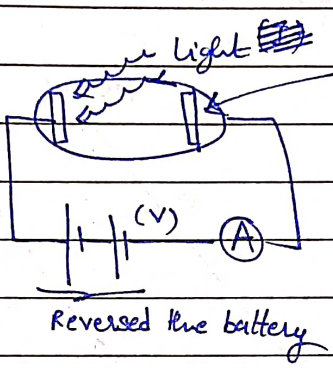


$$\boxed{K_{max} = (h\nu - \phi)}$$

V-I characteristic Curve :



Basically,



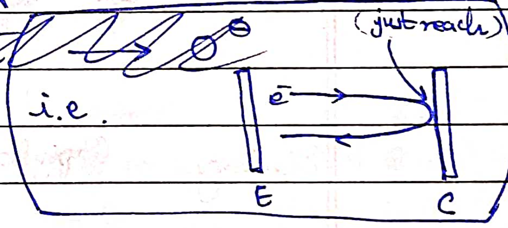
(only e^-s reaching here are those with energy > $e \cdot V$)

(as e^- : ⊕ ⟶ ⊖ So, it gain PE ⇒ lose KE)

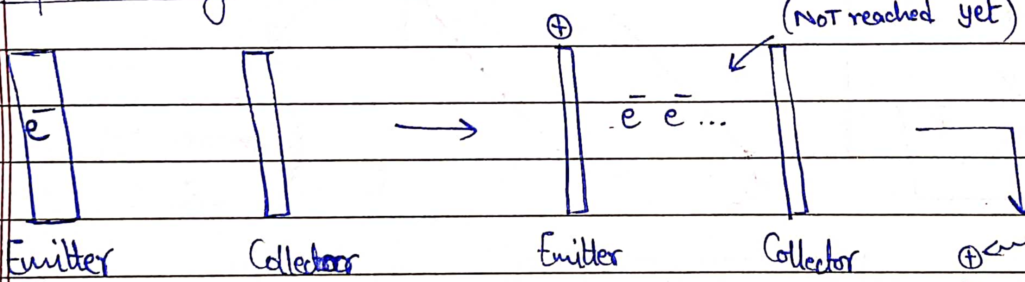
Here,
$$\boxed{eV_0 = (h\nu - \phi)}$$

At stopping potential,
$$\boxed{KE_{final} = 0}$$

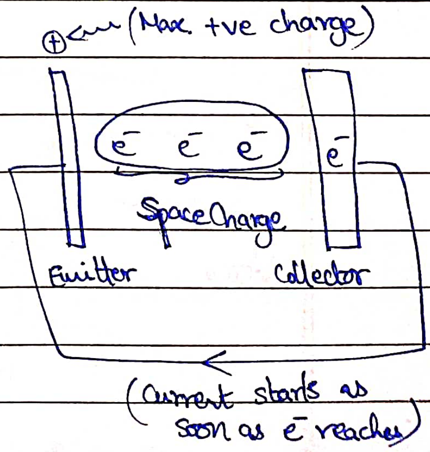
☆ ~~At stopping potential, total energy of e^- ⟶ 0~~



Space Charge :



$$\boxed{\left(\text{Max (+ve) charge on Emitter}\right) = \left(\text{Space Charge}\right)}$$



Thus, emitter has a ⊕ charge & attracts e^-. So, we need a certain min. (+ve) V to ensure saturation current.

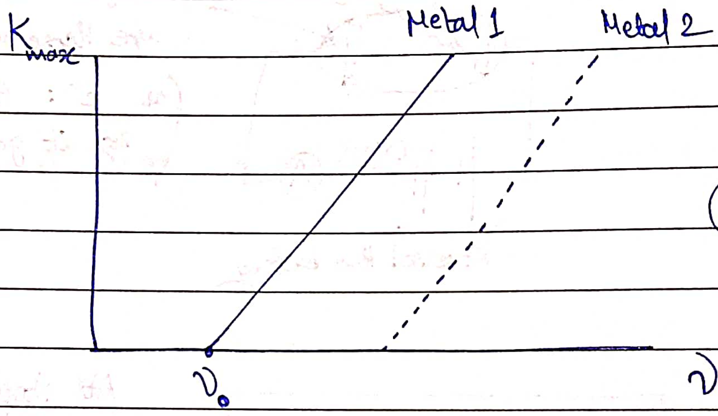
Now, saturation current = Const. as Max. current

when (No. of e^- leaving Emitter) = (No. of e^- reaching Collector)

having MORE speed

If we inc. V, e^- will reach collector ~~faster~~

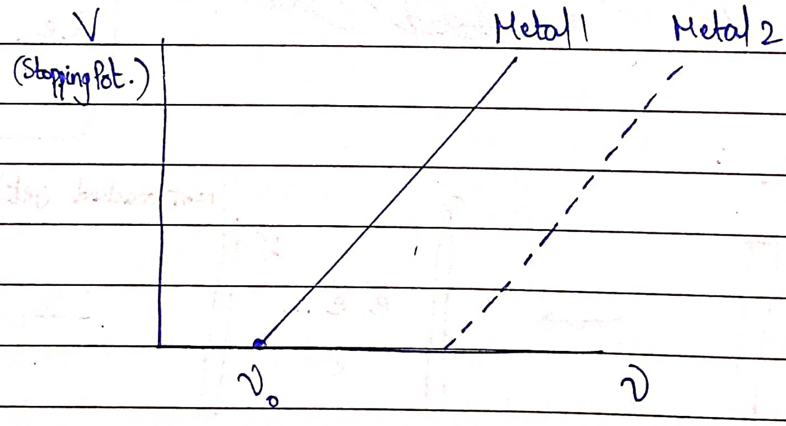
But (no. of e^- per sec. = Same) $\Rightarrow$ Current same.



$K_{max} = (h\nu - h\nu_0)$

Graphs for dif. Metals are Parallel lines

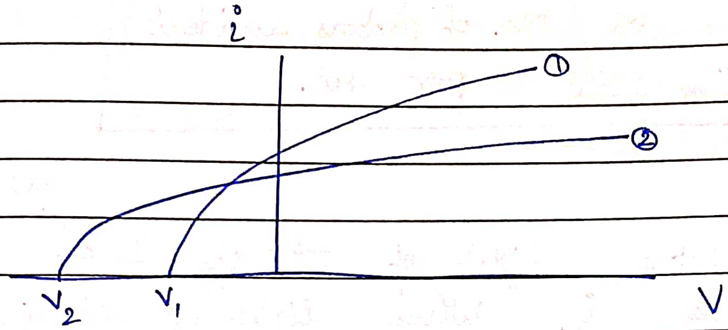
Similary,



$$\boxed{V_0 = \frac{h}{e}(\nu - \nu_0)}$$

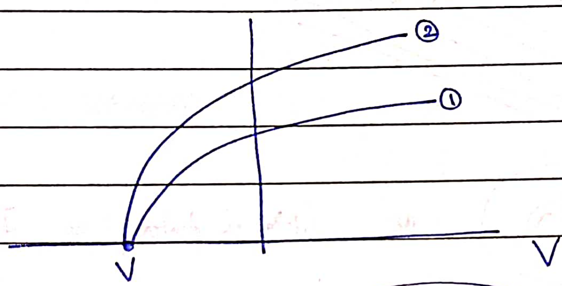
Notice, in both graphs $\boxed{\text{(Slope of Graph) is Const.}}$

Now,



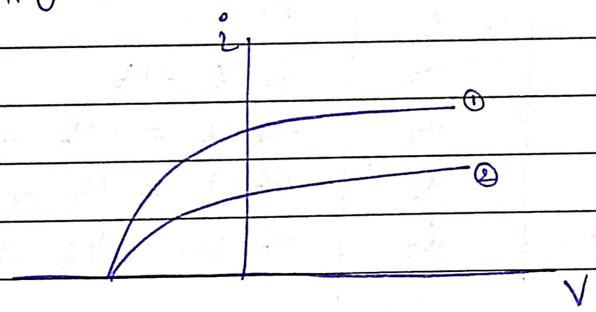
$V_1 \neq V_2$ (Stopping Pot.) $\Rightarrow$ Diff. Metals

And,



$V_1 = V_2$ (Stopping Pot.) $\Rightarrow$ Same Metal

Now,



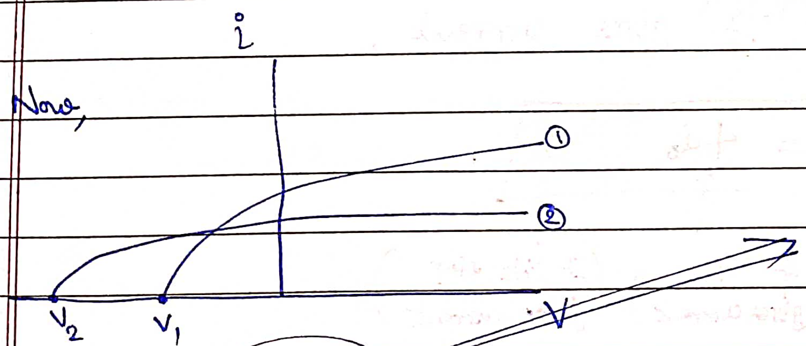
(No. of photon/e⁻ per sec in 1) > (No. of photon/e⁻ per sec. in 2)

$\Downarrow$ Since same metal

$\boxed{I_1 > I_2}$

☆ Assuming all photons release an e⁻.

Now,



$V_2 > V_1$ (Stopping Pot.) $\Rightarrow$ $E_2 > E_1$ (Energy of 1 photon) but $i_1 > i_2$ (sat. current) $\Rightarrow$ $N_1 > N_2$ (No. of photons/e⁻ per sec)

$\Rightarrow$ $E_2 N_2$ vs $E_1 N_1$ CAN'T compare

Now, $\boxed{i_{sat.} \propto \left(\text{No. of photons incident per sec.}\right)}$

Q) In a photo's setup, light of intensity I_0 and freq. ν_0 is incident. Sat. current was i_0. When light of intensity $2I_0$ and ν_0, sat. current was i_1. When light of intensity $2I_0$ and $\nu_0/2$, sat. current was i_2. Find i_1 and i_2, i_3. When light $2I_0$ and $2\nu_0$, sat. current was i_3.

~~A) $i \propto$ (No. of photons incident) $\Rightarrow$ i~~

A) (No. of photons) $\propto I/\nu_0$ as $\text{(No. of photons)} = \dfrac{IA\cos(\theta)}{E}$ ← E (Energy of 1 photon)

and $E = h\nu$.

So,

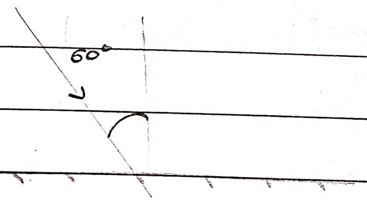
I	ν	i	N
I_0	ν_0	i_0	$I_0/\nu_0 = N_0$
$2I_0$	ν_0	i_1	$2I_0/\nu_0 = 2N_0 \Rightarrow i_1 = 2i_0$
$2I_0$	$2\nu_0$	i_2	$2I_0/2\nu_0 = N_0 \Rightarrow i_2 = i_0$
$2I_0$	$\nu_0/2$	i_3	$2I_0/(\nu_0/2) = 4N_0 \Rightarrow$?

If ν_0 gives current $\Rightarrow$ $2\nu_0$ gives current

$\not\Rightarrow$ $(\nu_0/2)$ gives current.

So, $\boxed{i_3 = 4i_0 \ , \ 0}$

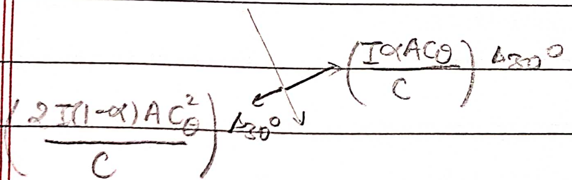
($4i_0$: If $\nu_0/2$ gives current) (0: If $\nu_0/2$ NOT gives current)

Q. 

Net force exerted by light makes 30° from normal. Find absorptivity of surface.

A.

$\frac{2I(1-\alpha)AC_\theta^2}{c}$  $\frac{I\alpha A C_\theta}{c}$



$\left(\frac{2I(1-\alpha)AC_\theta^2}{c}\right) S_{30°} = \left(\frac{I\alpha AC_\theta}{c}\right) S_{30°}$

$\Rightarrow 2(1-\alpha)\underbrace{C_\theta}_{\frac{1}{2}} = \alpha \quad (\theta = 60°)$

$\Rightarrow \underline{\alpha = 1/2}$

Doppler Effect in Light

$$f = f_0\sqrt{\frac{1-v/c}{1+v/c}}$$

~~(Approach)~~
(separation)

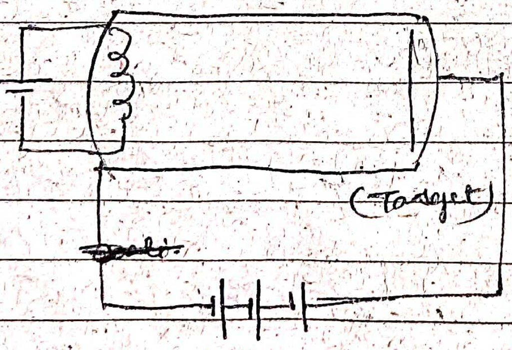
$v \rightarrow$ (Rel. vel b/w observer & source)

$$f = f_0\sqrt{\frac{1+v/c}{1-v/c}}$$

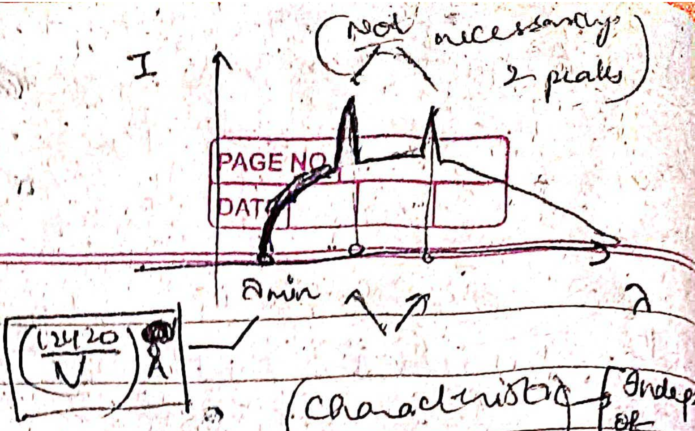
(Approach)

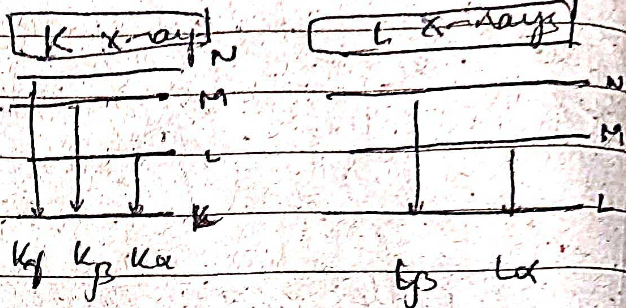
• X-RAYS

Reverse effect of Photo-e⁻ effect.
(Coolidge Tube)



e^- hit target ⇒ become photons (X-Rays)





Char X rays – produced by e^- transition due to vacancy created due to removal of e^- ~~by external field~~ in target atoms ~~by~~ ~~from~~

$E(K_\alpha) < E(K_\beta) < E(K_\gamma)$

$E(K_\alpha) > E(L_\alpha) > E(M_\alpha)$

$E \propto f \propto \frac{1}{\lambda}$ ⇒ same rel^n hold for f & opp for λ.

also, $E(K_\beta) = E(K_\alpha) + E(L_\alpha)$

$\Rightarrow f(K_\beta) = f(K_\alpha) + f(L_\alpha)$

$\Rightarrow \frac{1}{\lambda(K_\beta)} = \frac{1}{\lambda(K_\alpha)} + \frac{1}{\lambda(L_\alpha)}$

$E(K_\gamma) = E(K_\alpha) + E(L_\alpha) + E(M_\alpha)$

$= E(K_\alpha) + E(L_\beta)$

$= E(K_\beta) + E(M_\alpha)$

→ Moseley's law –

$$\boxed{\sqrt{f} = a(Z-b)}$$

(Atomic no)

(Characteristic X-ray) (Characteristic consts) (depends on series being observed)

* if f is of same type.

i.e K_α for all elems, K_β for all elems. etc.

For K_α, $\boxed{a = \sqrt{\frac{3RC}{4}}}$ (Rydberg const)

& for entire K series, $\boxed{b = 1}$

Proof:–

$$\frac{hc}{\lambda} = Rhc\left(1 - \frac{1}{n^2}\right)Z^2$$

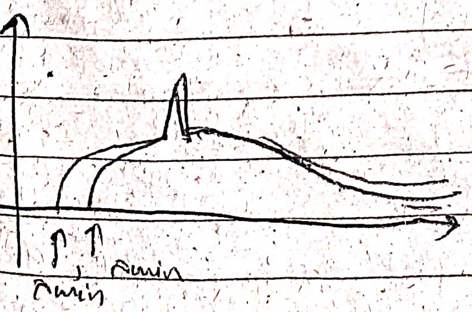
$$\Rightarrow f = RcZ^2\left(1 - \frac{1}{n^2}\right)$$

K_α: $n = 2$

$$\Rightarrow f = RcZ^2\left(\frac{3}{4}\right)$$

$$\Rightarrow \sqrt{f} = \sqrt{\frac{3RC}{4}}\, Z$$

$$\approx \sqrt{\frac{3RC}{4}}\,(Z-1)$$



$V' > V \Rightarrow \lambda'_{min} < \lambda_{min}$

BOHR'S MODEL (of H like atoms)

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→ Rutherford's Gold leaf Experiment

Nucleus – All +ve charge – Almost all mass

(e^- orbit nucleus)

(Mostly empty space)

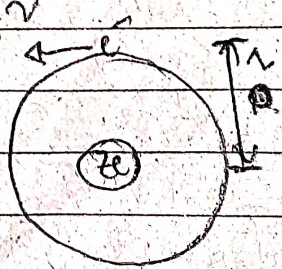
Limit – Couldn't explain stability of atom. Acc. charges radiate energy.

→ Bohr's Model

(1) Bohr's Quantization condⁿ $\boxed{mvr = n\hbar}$, $\hbar = \frac{h}{2\pi}$

No loss of energy when e^- in these orbits.

$$\frac{mv^2}{r} = \frac{kZe^2}{r^2}$$



$$P.E = -\frac{kZe^2}{r}$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{kZe^2}{r}\right) = \frac{kZe^2}{2r}$$

$$T.E = P.E + KE = -\frac{kZe^2}{2r}$$

$\frac{mv^2}{r} = \frac{kZe^2}{r^2}$, $mv = \frac{nh}{r}$

$\Rightarrow m = \frac{m^2v^2}{mv^2} = \frac{n^2h^2}{kZe^2} \frac{\cancel{r}}{r\cancel{}}$

$r = \frac{n^2h^2}{kmZe^2}$

$$\boxed{r = \frac{h^2}{mke^2}\left(\frac{n^2}{Z}\right)}$$

(2)

$r = r_0\left(\frac{n^2}{Z}\right)$

$E = \frac{-kZe^2}{2}\left(\frac{mke^2}{h^2}\right)\left(\frac{Z}{n^2}\right)$

$= \boxed{\frac{m}{2}\left(\frac{ke^2}{h}\right)^2\left(\frac{Z}{n}\right)^2} = -13.6\left(\frac{Z}{n}\right)^2 eV/atom$

$\frac{1}{2}mv^2 = KE = \frac{m}{2}\left(\frac{ke^2}{h}\right)^2\left(\frac{Z}{n}\right)^2$

$\Rightarrow \boxed{v = \left(\frac{ke^2}{h}\right)\left(\frac{Z}{n}\right)}$

Q. A hypothetical particle whose mass & charge are both 2x that of e^- orbits around a fixed proton & follows Bohr's quantization rule. Find ground state energy of this hyp particle

A. $\frac{E'}{E} = \left(\frac{m'}{m}\right)\left(\frac{q'}{e}\right)^2 = 2 \times 2^2 = 8$

only e^2 due to e^- & rest e^2 due to proton

Q. $\dot{M}$ (fixed) or (Very massive) m follows Bohr's quantization rule. Find ground state energy

A. $Ze \to M$

$e \to m$

$k \to G$

$E = -\left(\frac{m}{2}\right)\left(\frac{GMm}{h}\right)^2\left(\frac{1}{n}\right)^2$

OR (using traditional method)

$E_n = \frac{-13.6}{n^2}$ eV

Energy of Hydrogen's n^{th} state

$= \frac{-13.6}{n^2}\frac{Z^2}{Z^2}$ eV

$= \frac{-13.6 Z^2}{(nZ)^2}$ eV

⇓

Energy of

n^{th} state in ryd	=	$(nZ)^{th}$ state in ryd like Z ion

⇓

Energy of

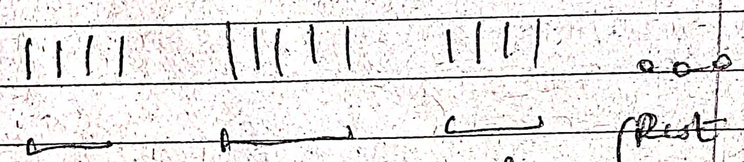
m ↓ n (in H) = Zm ↓ Zn (in Z ion)

* All hyd. like ion spectrum contain spectrum of hyd.

α line — $(n+1)^{th} \rightarrow n^{th}$
(First line)
Min E & f
Max λ

*Q. If visible light of λ 400-780 nm passes through H_2 gas, which lines observed in absorption spectrum

A. None absorbed since min energy for ① → ② not present.



Lyman $n \rightarrow 1$ (UV)

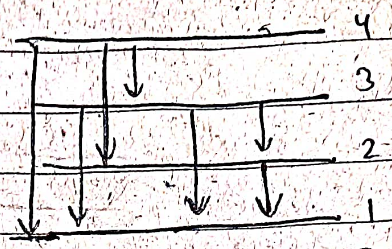
Balmer $n \rightarrow 2$ (First 4 lines in Visible, Rest UV)

Paschen $n \rightarrow 3$ (All IR)

(Rest overlap)

Q. All ions of H like gas are in particular excited state with principal quantum no. 'n'?. In its emission spectrum, first line of (Lyman series of H) along with 5 other spectral lines are observed. Find n & recognise ion

A.



6

Also $\underbrace{\frac{n(n-1)}{2}}_{\text{Total lines}} = 6 \Rightarrow \underline{n=4}$

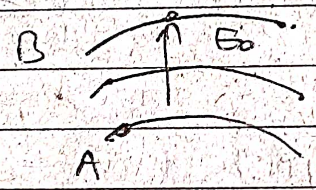
H: $2 \rightarrow 1$

Z: $2Z \rightarrow Z$, $2Z \le 4 \Rightarrow \boxed{Z \le 2}$

⇓

$\underline{Z = He^+}$

Q. All atoms of hyd gas are in particular state A. Absorbs E_0 → A → B. In emission spectrum, 6 diff types of spectral lines observed. ~~E_0~~ If some photons have $E > E_0$, $E = E_0$ & $E < E_0$.



Find n_A & n_B.

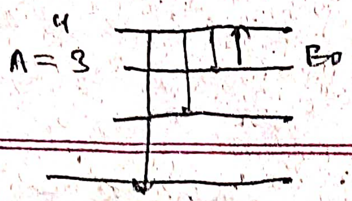
A. $\underline{n_B = 4}$ $\left(\frac{n(n-1)}{2} = 6\right)$
$\Rightarrow \underline{n=4}$

To check A,

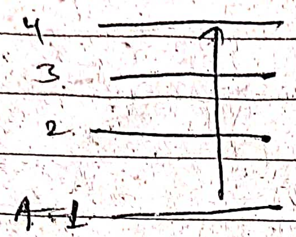
$A = 1, 2, 3$
$\quad\;\; (X) \;\; (X)$

$\Downarrow$

$\boxed{A=2}$



(No photon $\leq E_0$ energy)



(No photon energy) $\geq E_0$

Since at atomic scale, loss of energy only through EM wave.

(such as ~~emty~~ excitation of e^-

$\Rightarrow K \geq 2(E_1 - E_2)$)

(But this will not guarantee inelastic collision since energy obtained by e^- must be equal to some excitation energy)

Q. A neutron travelling with $KE = K$ eV strikes ground state H-atom at rest. Find min K s.t. the collision may be inelastic.
(Assume mass of Hyd atom & neutron are equal)

A

$\underset{m}{O} \xrightarrow{v} \quad \underset{m}{O}$

$$\text{Max loss} = \frac{1}{2}\left(\frac{m^2}{2m}\right)v^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$$

$$= \left(\frac{K}{2}\right)$$

if $\frac{K}{2} < \cancel{E_0}$ $(E_1 - E_2) \Leftarrow$ (Min excitation energy)

$\Downarrow$

Inelastic collision not possible

Q. $\overset{N}{O} \rightarrow 65\text{eV}$ (M_N) $\quad \overset{(4M_N)}{O}\, He$ (ground state)

if trajectory of N deflected by 90°, find possible KE of N after collision.

A

08/10/2023

NUCLEAR PHYSICS

- α, β, γ decay
- Nuclear force
- Binding Energy Curve
- Fission & Fusion
- Radioactivity

→ Decay

• α – $^{A}_{Z}X \to {}^{(A-4)}_{(Z-2)}X + {}^{4}_{2}He$ + Energy

(α-particle)

He

~~α-particle~~

★ Energy of all α-particles coming from same nucleus/atom is same.

• β – $n \to p + e^- + \bar{\nu}_e$

(-ve β decay)

(Anti-e⁻ neutrino)

(β-particle)

$\boxed{\text{Req. to conserve Spin, Lin. Mom \& Energy}}$

Qty conserved in nuclear reaⁿs:

1. Charge
2. Angular momentum (spin)
3. Linear momentum
4. # nucleons :- (protons + neutrons)
5. Mass-Energy

$n \to p + e^- + \bar{\nu}_e$

$(\frac{1}{2})\ (\frac{1}{2})\ (\frac{1}{2})\ (\frac{1}{2})$

$^{A}_{Z}X \to {}^{A}_{(Z+1)}Y + {}^{0}_{-1}e + \bar{\nu}_e$

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★ Energy of β-particles coming from same nucleus may have diff. energy

Reason: $\bar{\nu}_e$ & e^- can interact with e^- of the atom.

(Continuous graph)

• +ve β decay – $p \to n + e^+ + \nu_e$

(Positron)

(e^- neutrino)

★ Neutrino & Antineutrino only differ by sign of spin.

★ e^- & e^+ only differ by sign of charge.

Since $m_{products} > m_{reactants}$

⇓

Energy of proton ↑

⇓

only excited proton gives +β decay.

Outside nucleus, ~~proton~~ p stable.

But n unstable.

$t_{half} = 660$ sec

$$^{A}_{Z}X \rightarrow {}^{A}_{(Z+1)}Y + {}^{0}_{-1}e + \bar{\nu}_e$$

- n/p ratio — Determines stability

- γ decay

$$^{A}_{Z}X \rightarrow {}^{A}_{Z}X + \gamma \quad (1 \text{ or } 0)$$

Nucleus comes from higher energy state to lower energy state

From nuclei, (α, γ), (β, γ) (only γ) can be emitted.

(α, β) cannot be emitted together.

Decay of γ — Pair production

$$\gamma \rightarrow e^- + e^+$$

Cond^n: $\gamma \rightarrow e^+ + e^-$

1.02 MeV → e^+: 0.51 MeV, e^-: 0.51 MeV (rest energies)

(Min. energy req. for pair production)

★ Linear momentum not conserved since γ not at rest but e^+ & e^- at rest.

So, for pair prod^n, γ must transfer its momentum to a heavy nucleus at the instant it happens.

- Pair annihilation

$$e^+ + e^- \rightarrow 2\gamma$$

(combine at rest) (To conserve linear momentum)

$\leftarrow \gamma \quad \gamma \rightarrow$

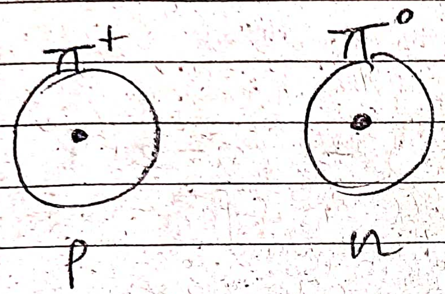
→ Nuclear force

• Yukawa's Meson Theory
↓
(Elementary particle)

π meson — (+ve), (−ve), neutral

π^+, π^-, π^0

n & p made of identical core.



Diff. is ~~...~~ π meson

Exchange of π mesons causes force, called exchange force

① Nuclear force is exchange force

② $F_{pp} = F_{nn} = F_{np}$ ← (Nuclear force)

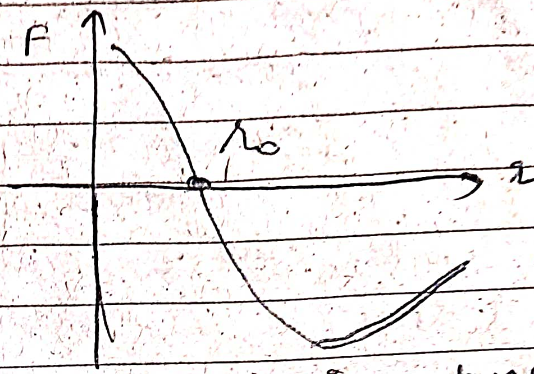
③ Charge independent

Net force: $F_{pp} < F_{nn} = F_{np}$.
(due to contribution of electrostatic force)

④ Short range

⑤ Mostly attractive force

If sep. b/w nucleon < 0.5 fermi (fm)
then nuclear force becomes highly repulsive



⑥ Strongest force in nature

⑦ Spin dependent.

Strong > Electromagnetic > Weak > Gravity

→ Nucleus

Radius – $R = R_0 (A)^{1/3}$

R_0 (Radius of Hyd nucleus) $= 1.1\ fm = 10^{-3} m$

A (Mass no.)

Density – $\rho = \frac{M}{V} = \frac{A\ amu}{\frac{4\pi}{3} R_0^3 A}$

$= \frac{3}{4\pi R_0^3}$

$\rho = 10^{12}\ kg/m^3$ → (All nucleus have same density)

→ **Binding Energy**

• **Mass defect** –

(Mass of nucleus) < (Total mass of nucleons)

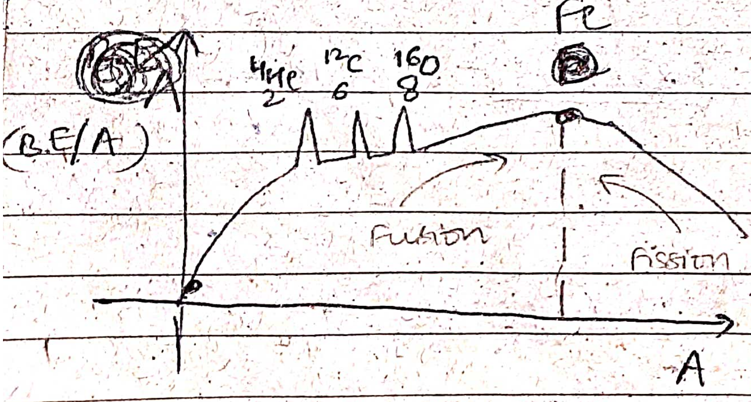
If all nucleons at ∞. To bring them together, work must have been done against $F_{electromag}$ to bring them together. This results in loss of mass-energy.

$$\boxed{\Delta m = Z m_p + (A-Z) m_n - m\left({}^{A}_{Z}X\right)}$$

• Binding energy (BE)

$$\boxed{BE = \Delta m c^2}$$

1 amu = 931.5 MeV



(Binding Energy Curve)

First ↑, then ↓

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Binding energy curve gives stability of nuclei.

$$\left(\frac{B.E}{A}\right)\uparrow \Rightarrow \text{stability} \uparrow$$

$\Downarrow$

(Iron is most stable element in nature)

${}^{4}_{2}He$ – $n=2$, $p=2$

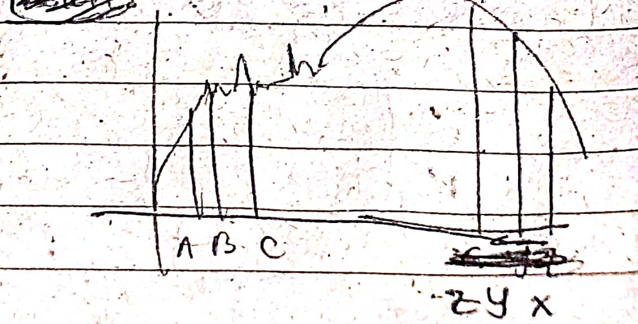
${}^{12}_{6}C$ – $n=6$, $p=6$

${}^{16}_{8}O$ – $n=8$, $p=8$

even-even nuclei

e-e nuclei are more stable than neighbouring nuclei.

$$\boxed{e\text{-}e > e\text{-}o > o\text{-}o}$$



The Liquid-Drop Model

The **liquid-drop model,** first proposed in 1928 by the Russian physicist George Gamow and later expanded on by Niels Bohr, is suggested by the observation that all nuclei have nearly the same density. The individual nucleons are analogous to molecules of a liquid, held together by short-range interactions and surface-tension effects. We can use this simple picture to derive a formula for the estimated total binding energy of a nucleus. We'll include five contributions:

1. We've remarked that nuclear forces show *saturation;* an individual nucleon interacts only with a few of its nearest neighbors. This effect gives a binding-energy term that is proportional to the number of nucleons. We write this term as C_1A, where C_1 is an experimentally determined constant.
2. The nucleons on the surface of the nucleus are less tightly bound than those in the interior because they have no neighbors outside the surface. This decrease in the binding energy gives a *negative* energy term proportional to the surface area $4\pi R^2$. Because R is proportional to $A^{1/3}$, this term is proportional to $A^{2/3}$; we write it as $-C_2A^{2/3}$, where C_2 is another constant.

3. Every one of the Z protons repels every one of the $(Z - 1)$ other protons. The total repulsive electric potential energy is proportional to $Z(Z - 1)$ and inversely proportional to the radius R and thus to $A^{1/3}$. This energy term is negative because the nucleons are less tightly bound than they would be without the electrical repulsion. We write this correction as $-C_3 Z(Z - 1)/A^{1/3}$.
4. To be in a stable, low-energy state, the nucleus must have a balance between the energies associated with the neutrons and with the protons. This means that N is close to Z for small A and N is greater than Z (but not too much greater) for larger A. We need a negative energy term corresponding to the difference $|N - Z|$. The best agreement with observed binding energies is obtained if this term is proportional to $(N - Z)^2/A$. If we use $N = A - Z$ to express this energy in terms of A and Z, this correction is $-C_4(A - 2Z)^2/A$.
5. Finally, the nuclear force favors *pairing* of protons and of neutrons. This energy term is positive (more binding) if both Z and N are even, negative (less binding) if both Z and N are odd, and zero otherwise. The best fit to the data occurs with the form $\pm C_5 A^{-4/3}$ for this term.

The total estimated binding energy E_B is the sum of these five terms:

$$E_B = C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z - 1)}{A^{1/3}} - C_4 \frac{(A - 2Z)^2}{A} \pm C_5 A^{-4/3} \tag{19.4}$$

(nuclear binding energy)

The constants C_1, C_2, C_3, C_4, and C_5, chosen to make this formula best fit the observed binding energies of nuclides, are

$$C_1 = 15.75 \text{ MeV}$$
$$C_2 = 17.80 \text{ MeV}$$
$$C_3 = 0.7100 \text{ MeV}$$
$$C_4 = 23.69 \text{ MeV}$$
$$C_5 = 39 \text{ MeV}$$

The constant C_1 is the binding energy per nucleon due to the saturated nuclear force. This energy is almost 16 MeV per nucleon, about double the *total* binding energy per nucleon in most nuclides.

If we estimate the binding energy E_B using Eq. (19.4), we can solve Eq. (19.3) to use it to estimate the mass of any neutral atom:

$$ {}^{A}_{Z}M = ZM_H + Nm_n - \frac{E_B}{c^2} \quad \text{(semiempirical mass formula)} \tag{19.5}$$

Equation (19.5) is called the *semiempirical mass formula*. The name is apt; it is *empirical* in the sense that the C's have to be determined empirically (experimentally), yet it does have a sound theoretical basis.

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$A \rightarrow B + C$

(Fission)

OR

$B + C \rightarrow A$

(Fusion)

Energy released (Q-value of reaction) $= (\text{mass}_{LHS} - \text{mass}_{RHS})\, c^2$

$= (B.E_{RHS} - B.E_{LHS})\, c^2$

Q-val $< 0 \iff$ Exothermic (α, β^-, γ)

Q-val $> 0 \iff$ Endothermic (β^+, K-capture)

NOTE: $p \Rightarrow n + e^+ + \nu$

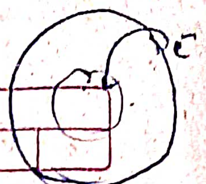
Reaction only possible when

$$\boxed{m_{\text{End Product}} - m_{\text{Initial Reactant}} > 2(m_{e^-})}$$

Since $m_n > m_p + m_e$

- K-capture – nucleus captures an e^- of K-shell

$p^+ + e^- \rightarrow n + \nu$



After K-capture, X-ray (α-series) is emitted.

→ Radioactivity

Inherent process.

Indep. of ext. parameters such as P, T.

So, uncontrolled process.

$A = \left(-\frac{dN}{dt}\right)$ → Activity (Rate of decay per sec)

[Since N ↓]

N – (# Active nuclei)

$\frac{dN}{dt} \propto N$

$\Rightarrow \boxed{\frac{dN}{dt} = -\lambda N}$

λ – (Decay const)

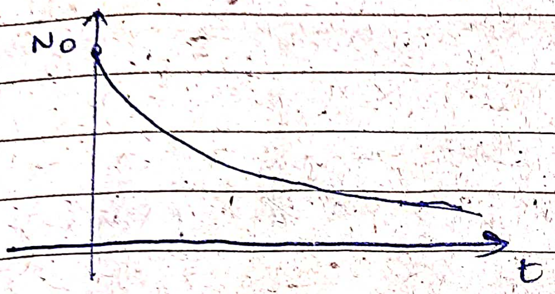
units: $1\,Ci = 3.7 \times 10^{10}$ /sec $= 3.7 \times 10^{10}$ Bq

(curie) (Becquerel)

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_0^t \lambda \, dt$$

$\Rightarrow \boxed{N = N_0 e^{-\lambda t}}$

Decay Rule

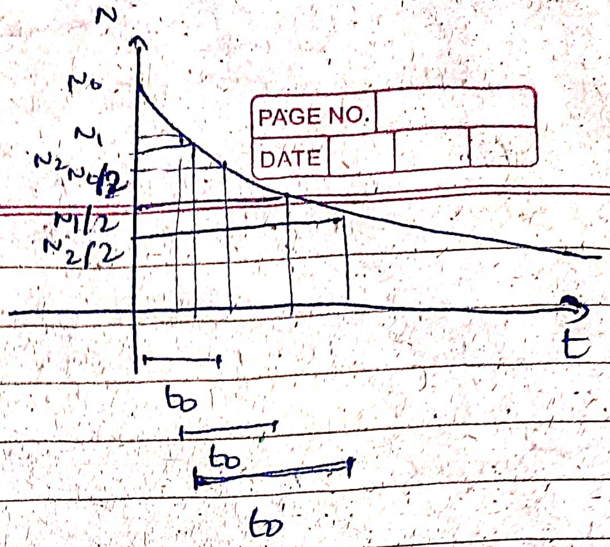


<u>Half life</u> – Time in which #active nuclei become half.

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}} \Rightarrow t_{1/2} = \frac{\ln(2)}{\lambda} = \boxed{\frac{0.693}{\lambda}}$$

NOTE: This result holds true for large enough Ns.

Each nuclei has 80% probability of decaying or <u>not</u> decaying in a $t_{1/2}$. Hence, if we have countable N, we <u>cannot</u> claim with certainty that after $t_{1/2}$, $\frac{N}{2}$ will remain only.



<u>Q.</u> A radioactive sample decays 10% in 1 hr. What fraction of initial sample will remain undecayed after 3 hrs.

<u>A</u>

<u>Q.</u> A container contains a mix of 2 radioactive samples A & B with $\lambda_A = 2\lambda_B$.

$$\frac{A_0(A)}{A_0(B)} = 10.$$

After what time the ratio their activities will become 1/10?

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A. $\dfrac{A_{0(A)}}{A_{0(B)}} = \dfrac{\lambda_A N_{0(A)}}{\lambda_B N_{0(B)}}$

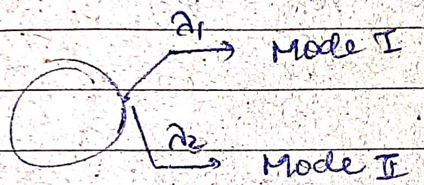
$$\frac{A_{(A)}}{A_{(B)}} = \frac{\lambda_A N_{(A)}}{\lambda_B N_{(B)}} = \frac{A_{0(A)}\, e^{-\lambda_A t}}{A_{0(B)}\, e^{-\lambda_B t}}$$

$$\Rightarrow \frac{1}{10} = 10\, e^{(\lambda_B - \lambda_A)t}$$

$$\Rightarrow (\lambda_B - \lambda_A)t = -2\ln(10)$$

$$\Rightarrow t = \frac{2\ln(10)}{\lambda_B}$$

- Simultaneous Decay (Parallel)



$$\boxed{\lambda_{eq} = \lambda_1 + \lambda_2} \Rightarrow \boxed{\frac{1}{t_{1/2}} = \frac{1}{T_1} + \frac{1}{T_2}}$$

(Half-life in modes separately)

Q. [V_0] A_0, λ
Soln leaks
$\dfrac{dV}{dt} = -\alpha V$

Find A_t.

A. Equivalent to simultaneous decay.

$$\frac{dN}{dt} = \underbrace{-\lambda N}_{\text{Due to radioact}} - \underbrace{\frac{N\,dV}{V}}_{\text{Due to leaking}} = -(\lambda + \alpha)N$$

$\lambda_{eq} = (\alpha + \lambda) \Rightarrow A = A_0 e^{-(\lambda+\alpha)t}$

Q. Radioactive sample (λ) is being prepared in lab at a const. rate α. Assume prepn starts at $t=0$. Find N in lab after time t. Hence, find N_{max}

A. $\dfrac{dN}{dt} = \alpha - \lambda N$

$\Rightarrow \cancel{\dfrac{dN}{dt} + \lambda N = \alpha}$

$\Rightarrow \cancel{[e^{\lambda t} N] = \int e^{\lambda t} \alpha\, dt}$

$\cancel{e^{\lambda t} N}$

$$\boxed{N = \frac{\alpha}{\lambda}\left[1 - e^{-\lambda t}\right]}$$

$$\left\{ \frac{dN}{dt} + \lambda N = \alpha \equiv R\frac{dq}{dt} + \frac{q}{C} = E_0 \right\}$$

- Series Decay

$$A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C$$

$N_0 = N_A + N_B + N_C$

$\Rightarrow N_C = N_0 - (N_A + N_B)$

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Q. $A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C$

at $t=0$, $N_A = N_0$,

$N_B = N_C = 0$.

find N_A, N_B, N_C as a f^n of time.

A $\boxed{N_A = N_0 e^{-\lambda_1 t}}$

$$\frac{dN_B}{dt} = \lambda_1 N_A - \lambda_2 N_B.$$

$$\Rightarrow \frac{dN_B}{dt} + \lambda_2 N_B = \lambda_1 N_0 e^{-\lambda_1 t}$$

$$I.F. = e^{\int \lambda_2 dt} = e^{\lambda_2 t}$$

$$\Rightarrow [e^{\lambda_2 t} N_B] = \int \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t} dt$$

$$\Rightarrow e^{\lambda_2 t} N_B = \frac{\lambda_1 N_0 e^{(\lambda_2-\lambda_1)t}}{(\lambda_2 - \lambda_1)} + C$$

at $t=0$, $N_B = 0$ $\Rightarrow$ $C = -\frac{\lambda_1 N_0}{(\lambda_2 - \lambda_1)}$

$$\Rightarrow N_B = \left(\frac{\lambda_1}{\lambda_2-\lambda_1}\right) N_0 e^{-\lambda_1 t} - \left(\frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\right) e^{-\lambda_2 t}$$

$$\boxed{= \left(\frac{\lambda_1}{\lambda_2 - \lambda_1}\right) N_0 \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)}$$

- Carbon dating.

$A \rightarrow$ (Activity of C^{14} in dead sample)

$A_0 \rightarrow$ (Activity of C^{14} in live sample)

$$\boxed{A = A_0 e^{-\lambda t}}$$